8.4 Limitations of quantum operations formalism

Are there interesting quantum systems whose dynamics are not described by quantum operations?

Suppose a single qubit is prepared in some unknown quantum state, which we denote ρ. The preparation of this qubit involves certain procedure to be carried out in the laboratory in which the qubit is prepared.

Suppose

$$\rho \otimes |0\rangle\langle 0| \otimes \text{other degrees of freedom} \tag{8.186}$$

if ρ is a state on the bottom half of the Bloch sphere, and

$$\rho \otimes |1\rangle\langle 1| \otimes \text{other degrees of freedom} \tag{8.187}$$

if ρ is a state on the top half of the Bloch sphere.

Once the state preparation is done, the system begins to interact with the environment, in this case all the laboratory degrees of freedom. Suppose the interaction is such that a c-not is performed between the principal system and the extra qubit in the laboratory system. Thus, if the system's Bloch vector was initially in the bottom half of the Bloch sphere it is left invariant by the process, while if it was initially in the top half of the Bloch sphere it is rotated into the bottom half of the Bloch sphere.

A quantum system which interacts with the degrees of freedom used to prepare that system after the preparation is complete will in general suffer a dynamics which is not adequately described within the quantum operations formalism.

It is an interesting problem for further research to study quantum information processing beyond the quantum operations formalism.