8.4 Applications of quantum operations

The quantum operations formalism has numerous applications

8.4.1 Master equations

Open quantum systems occur in a wide range of disciplines, and many tools other than other than quantum operations can be employed in their study.

The dynamics of open quantum systems have been studied extensively in the field of quantum optics. The main objective in this context is to describe the time evolution of an open system with a differential equation which properly describes non-unitary behavior.

This description is provided by the master equation, which can be written most generally in the *Lindblad form* as

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_j \left[2L_j\rho L_j{}^\dagger - \{L_j{}^\dagger L_j, \rho\}\right], \tag{8.134}$$

where $\{x,y\} = xy + yx$ denotes an anticommutator, H is the system Hamiltonian, a Hermitian operator representing the coherent part of the dynamics, and L_j are the *Lindblad operators*, representing the coupling of the system to its environment. $\rightarrow$

- The differential equation takes on the above from in order that the process be completely positive.
- It is also generally assumed that the system and environment begin in a product state.
- To derive a master equation for a process, one usually begins with a system-environment model Hamiltonian, and them makes the Born and Markov approximation in order to determine L_j.
- In the master equation approach, $\mathrm{tr}\,[\rho(t)] = 1$ at all times.

8.4.2 Quantum process tomography

How do quantum operations relate to experimentally measurable quantities?
What measurements should an experimentalist do if they wish to characterize the dynamics of a quantum system?
Classical system $\rightarrow$ system identification
Quantum open system $\rightarrow$ quantum process tomography (can be performed for finite dimensional quantum systems)

Quantum state tomography : the procedure of experimentally determining an unknown quantum state.

A single copy of ρ : it is impossible to characterize ρ (no cloning theorem; indistinguishability)
A large number of copies of ρ : it is possible to estimate ρ

Quantum process tomography : The experimental procedure which can determine the ensemble of output state (with d-dimension) using d^2-dimensional pure input states. $\rightarrow$ experimental procedure for understanding and controlling noisy quantum system.

Schrödinger's cat

When I hear about Schrödinger's cat, I reach for my gun
Schrödinger's infamous cat faces life or death contingent upon an automatic device which breaks a vial of poison and kills the cat if an excited atomic state is observed to decay: Schrödinger asked what happens when the atom is in a superposition state? **Is the cat alive or dead?** Why do superposition states such as this apparently not occur in the everyday world?

$$|\psi\rangle = \left[|\text{dead}\rangle|0\rangle + n|\text{alive}\rangle|1\rangle\right]/\sqrt{2},$$

in which the cat's state has become entangled with that of the atom. The density matrix of this state,

$$\rho = |\psi\rangle\langle\psi|$$

Now, in practice it is impossible to perfectly isolate the cat and the atom in their box, and thus information about this superposition state will leak into the external world.
eg) heat from the cat's body could permeate the wall and give some indication of its state to the outside. Such effects may be modeled as phase damping, which exponentially damps out the final two (off-diagonal) terms in ρ. To a first approximation, we may model the cat-atom system as a simple harmonic oscillator. An important result about the decoherence of such a system is that coherence between states of high energy difference decays faster than between states with a lower energy difference. Thus ρ will quickly be transformed into a nearly diagonal state, which represents an ensemble of cat-atom states which correspond to eigher dead or alive, and are not in a superposition of the two states.