Chapter 8

Quantum noise and quantum operations

Until now we have dealt almost solely with dynamics of closed quantum systems, that is, with quantum system that do not suffer any unwanted interactions with the outside world. However, real systems suffer from unwanted interactions with the outside world. These unwanted interactions show up as *noise* in quantum information processing systems. We need to understand and control such noise processes in order to build useful quantum information processing systems.

The third part begins with the description of the *quantum operation formalism*, a powerful set of tools enabling us to describe quantum noise and the behavior of *open* quantum systems.

The quantum operation formalism is very powerful, in that it simultaneously addresses a wide range of physical scenarios. It can be used to describe:

- nearly closed systems which are weakly coupled to their environment.
- systems which strongly coupled to their environment.
- closed systems that are opened suddenly and subject to measurement.

Another advantage of quantum operations in applications to quantum computation and quantum information is that they are especially well adapted to describe *discrete state changes*, that is, transformations between an initial state ρ and final state ρ', without explicit reference to the passage of time. This discrete-time analysis is rather different to the tools traditionally used by physicists for the description of open quantum systems (such as 'master equations', 'Langevin equations', and 'stochastic differential equations'), which tend to be continuous-time descriptions.

8.1 Classical noise and Markov processes

To understand noise in quantum systems it is helpful to build some intuition by understanding noise in classical systems.

Suppose p_0 and p_1 are the initial probabilities that the bit is in the states 0 and 1, respectively. Let q_0 and q_1 be the corresponding probabilities after the noise has occurred. Let X be the initial state of the bit, and Y the final state of the bit. Then the law of total probability states that

$$p(Y = y) = \sum_x p(Y = y|X = x)p(X = x). \tag{8.1}$$

The conditional probabilities $p(Y = y|X = x)$ are called *transition probabilities*, since they summarize the changes that may occur in the system.

$$\begin{bmatrix} q_0 \\ q_1 \end{bmatrix} = \begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \end{bmatrix}. \tag{8.2}$$

Imagine that we are trying to build a classical circuit to perform some computational task : a *Markov Process* $X \rightarrow Y \rightarrow Z$ (*independent* process *cf.* Markovian an non-Markovian). Physically, this assumption of Marvovicity corresponds to assuming that the environment causing the noise in the first NOT gate, a reasonable assumption given that the gates are likely to be located a considerable distance apart in space.

Noise in classical systems can be described using the theory of stochastic processes. Often, in the analysis of multi-stage processes it is a good assumption to use Markov processes. For a single stage process the output probabilities $\vec{q}$ are related to the input probabilities $\vec{p}$ by the equation

$$\vec{q} = E\vec{p}, \tag{8.3}$$

where E is a matrix of transition probabilities which we shall refer to as the *evolution matrix*. Thus, the final state of the system is linearly related to the starting state. This feature of linearity is echoed in the description of quantum noise, with density matrices replacing probability distributions.

If $\vec{p}$ is a valid probability distribution, then $E\vec{p}$ must also be a valid probability distribution. Thus, E must be satisfied

1. the positivity requirement (non-negative E)
2. completeness requirement (unity sum of E)