2.3 Application: superdense coding

Two parties, conventionally known as Alice and Bob, want to transmit some classical information. Suppose Alice has two classical bits of information which she wishes to send Bob, but is only allowed to send a single qubit to Bob.

Suppose Alice and Bob initially share a pair of qubits in the entangled state

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Alice is initially in possession of the first qubit, while Bob has possessionof the second qubit. Note that $|\psi\rangle$ is a fixed state(from Alice or third party).

She wishes to send the bits string;

- '00' : she does noting at all to her quibit.
- '01' : she applies the phase flip Z to her qubit.
- '10' : she applies the quantum NOT gate X.
- '11' : the she applies the iY gate to her qubit.

The four resulting states are easily seen to be:

$$\begin{aligned} 00 : |\psi\rangle &\rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}} \\ 01 : |\psi\rangle &\rightarrow \frac{|00\rangle - |11\rangle}{\sqrt{2}} \\ 10 : |\psi\rangle &\rightarrow \frac{|01\rangle + |10\rangle}{\sqrt{2}} \\ 11 : |\psi\rangle &\rightarrow \frac{|01\rangle - |10\rangle}{\sqrt{2}}. \end{aligned}$$

Then Alice sends her qubit to Bob, giving Bob possession of both qubits, then by doing a measurement in the Bell basis Bob can dertime which of the four possible bit string Alice sent. It means Alice, interacting with only a single qubit i, is able to transmit two bits of informtion to Bob.

Of course, two qubits are involved in the protocol, but Alice never need interact with the second qubit. Classically, the task Alice accomplishes would have been impossible had she only transmitted a single classical bit.