2.2 The postulates of quantum mechanics

The postulates of quantum mechanics were derived afeter a long process of trial and (mostly) error, which involved a considerable amount of guessing and fumbling by the orginators of the theory.

2.2.1 State space

What the arena in which quantum mechanics take place is.

> **Postulate 1**; Associated to any isolated physical system is a complex vector space with inner product (that is a Hilbert space) known as the *state space*of the system. The system is completely described by its *state vector*, which is a unit vector in the system's state space.

Qauntum mechanics does not tell us what the state space of that system is, nor that the space vector of the system is. Figuring that out for a specific system is a difficult problem. eg) **QED, ISING, HUBBARD model**

A general state can be said as a linear combination of unit vectors, that is

$$|\Psi\rangle = \sum_i \alpha_i |\psi_i\rangle$$

where $|\Psi\rangle$ is a general state, α_i is (probability) amplitude for unit state $|\psi_i\rangle$. We can say that a $|\Psi\rangle$ is the superposition of the states $|\psi_i\rangle$

2.2.2 Evolution

> **Postulate 2**: The evolution of a *closed* quantum system is described by a *unitary transformation*. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2
>
> $$|\psi'(t')\rangle = U(t', t)|\psi(t)\rangle.$$

Just as quantum mechanics does not tell us the state space or quantum state of a *particular* quantum system, it does not tell us which unitary operators U describe real-world quantum dynamics. Recall unitary operators described in Chapter 1. (the quantum NOT gate X, Z, and Hadamard gate)

Postulate 2 requires that the system being described be closed. (no interaction with other systems) In reality, all systems interact at least somewhat with other systems.Nevertheless, there are interesting systems which can be described as part of a larger closed system. **In principle, every open system can be described as part of a larger closed system which is undergoing unitary evolution.**(Ch.8)

Postulate 2′: The time evolution of the state of a closed quantum system is described by the *Schrödinger equation*,

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

In general figuring out the Hamiltonian needed to describe a particular physical system is very difficult problem which requires substantial input from experiment in order to be answered. Because Hermitonian is a Hermitian operator it has a spectral decomposition

$$H = \sum_E E|E\rangle\langle E|$$

with eigenvalues E(Energy) and corresponding normalized e-vectors $|E\rangle$: energy e-sts., stationary sts. The lowest energy: *ground state energy*
corresponding energy e-st. : *ground state*
The reason the states $|E\rangle$ are sometimes known as stationary states is because their only change in time is to acquire an overall numerical factor,

$$|E\rangle \rightarrow \exp\left(-iEt/\hbar\right)|E\rangle.$$

As an example, suppose a simgle qubit has Hamiltonian

$$H = \hbar\omega X.$$

The energy eigenstates of this Hamiltonian are obviously the same as the eigenstates fo X, $|+\rangle$ and $|-\rangle$, with coressponding energies $\hbar\omega$ and $-\hbar\omega$. The ground state is therefore $|-\rangle$ and ground state energy is $-\hbar\omega$.
What is the connection between Postulate $2'$, and Postulate 2?

$$|\psi(t2)\rangle = \exp\left[\frac{-iH(t_2 - t_1)}{\hbar}\right]|\psi(t_1)\rangle = U(t_1, t_2)|\psi(t_1)\rangle$$

where we define

$$U(t_1, t_2) \equiv \exp\left[\frac{-iH(t_2 - t_1)}{\hbar}\right]$$

Any operator U can be realized in the form $U = \exp(iK)$ for some Hermitian operator K. There is therefore a one-to-one correspondence between the discrete-time description of dynamics using unitary operators, and the continuous time description using Hamiltonians.

In quantum computation and quantum information we often speak of *applying* a unitary operator to a particular quantum system. $\Rightarrow$ Doesn't this contradict what we said earlier, about unitary operators describing the evolution of a *closed* quantum system? After all, if we are 'applying' a unitary operator, then that implies that there is an external 'we' who is interacting with the quantum system, and the is not closed.

An example of this occurs when a laser is focused an atom. After a lot of thought and hard work it is possible to write down a Hamiltonian describing the total atom-laser system. The behavior of the state vector of the atom turns out to be almost but not quite prefectly descibed by another Hamiltonian, the *atomic Hamiltonian.* The atomic Hamiltonian contains terms related to laser intensity, and other parameters of the laser. It is *as if* the evolution of the atom were begin described by a Hamiltonian which we can vary at will, despite the atom not being a closed system.

The upshot is that to gein we will often describe the evolution of quantum systems - even systems which aren't closed - using unitary operators. The main exception to this is <u>quantum measurement</u>.

2.2.3 Quantum measurement

The evolution of the closed system is well described unitary operator, but if the other system like experimentalist observes the system to find what is going on inside the system, an interaction makes the system no longer closed, and thus not necessarily subject to unitary evolution. Postulate 3 explain the effects of measurements on quantum systems,

Postulate 3: Quantum measuerments ared described by an collection M_m of *measurement operators*. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur int the experiment. If the state of the quantum system is $|\psi\rangle$ immediatedly before the measurement then the probability that result m occurs is given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle.$$

and the state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$$

The measuement operators satisfy the *completeness equation*,

$$\sum_m M_m^\dagger M_m = I.$$

The completeness equation expresses the fact that probabilities sum to one:

$$1 = \sum_m p(m) = \sum_m \langle\psi|M_m^\dagger M_m|\psi\rangle$$

The completeness equation is much easier to check directly, to that's why it appears in the statement of the postulate.
The *measurement of a qubit in the computational basis*:

$$\begin{aligned} M_0 &= |0\rangle\langle 0| \\ M_1 &= |1\rangle\langle 1|. \end{aligned}$$

Each measurement operator is Hermitian, and that $M_0^2 = M_0$, $M_1^2 = M_1$. Thus completeness relationis obeyed, $I = M_0^\dagger M_0 + M_1^\dagger M_1 = M_0 + M_1$. Suppose the state being measure is $|\psi\rangle = a|0\rangle + b|1\rangle$. Then the probability of obtaining measurement outcome 0 is

$$p(0) = \langle\psi|M_0^\dagger M_0|\psi\rangle = \langle\psi|M_0|\psi\rangle = |a|^2.$$

Similarly, $p(1) = |b|^2$. The state after measurement in the two cases is therefore

$$\begin{aligned} \frac{M_0|\psi\rangle}{|a|} &= \frac{a}{|a|}|\psi\rangle \\ \frac{M_1|\psi\rangle}{|b|} &= \frac{b}{|b|}|\psi\rangle \end{aligned}$$

The status of Postulate 3 as a fundamental postulate intrgues many people.
→ According to Postualte 2, the evolution of this larger isolated system can be described by a unitary evolution. Might it be possible to derive Postulate 3 as a consequence of this pircture? Despite considerable investigation along these lines there is still disagreement between physicists about whether or not this is possible.

2.2.4 Distinguishing quantum states

An important application of Postulate 3 is to the problem of *distinguishing quantum states.*

Classical world : distinct states of an object are usually distinguishable.
Quantum mechanics : non-orthogonal quantum states cannot be distinguished.

Alice choose $|\psi_i\rangle (1 \leq i \leq n)$ from some fixed set of states known to Alice and Bob. She gives state $|\psi_i\rangle$ to Bob, whose task is to identify the index i of the state Alice has given him.

1. If states $|\psi_i\rangle$ are orthonormal, the bob can do a quantum measurement to *distinguish* these states.

   Def. measurement operators $M_i \equiv |\psi_i\rangle\langle\psi_i|$, one for each index i
   An additional measurement operators M_0 defined as the positive square root of the positive operator $I - \sum_{i\neq 0} |\psi_i\rangle\langle\psi_i|$. These operators satisfy the completeness relation, and if the state $|\psi\rangle$ is prepared then $p(i) = \langle\psi_i|M_i|\psi_i\rangle = 1$, so the result i occurs with certainty, Thus, it is possible to reliably distinguish the orthonormal states $|\psi_i\rangle$

2. If the states $|\psi\rangle$ are not orthonormal then we can prove that there is *no quantum measurement capable of distinguishing the states.* → non-orthogonal states can't be reliably distinguished

   Suppose that measurement distinguishing the non-orthogonal states $|\psi_1\rangle$ and $|\psi_2\rangle$ is possible.
   If the state $|\psi_1\rangle(|\psi_2\rangle)$ is prepared then the probability of measuring j such that $f(j) = 1(f(j) = 2)$ must be 1. Defining $E_i \equiv \sum_{j:f(j)=i} M_j^\dagger M_j$, these observations may be written as:

   $$\langle\psi_1|E_1|\psi_1\rangle = 1; \langle\psi_2|E_2|\psi_2\rangle = 1$$

   Since $\sum_i E_i = I$ it follows that $\sum_i \langle\psi_i|E_i|\psi_i\rangle = 1$, and since $\langle\psi_1|E_1|\psi_1\rangle = 1$ we must have $\langle\psi_1|E_2|\psi_1\rangle = 0$, and thus $\sqrt{E_2}|\psi_1\rangle = 0$. Suppose we decompose $|\psi_2\rangle = \alpha|\psi_1\rangle + \beta|\phi\rangle$, where $|\phi\rangle$ is orthonormal to $|\psi_1\rangle$, $|\alpha|^2 + |\beta|^2 = 1$, and $|\beta| < 1$ since $|\psi_1\rangle$and $|\psi_2\rangle$ are not orthogonal. Then $\sqrt{E_2}|\psi_2\rangle = \beta\sqrt{E_2}|\phi\rangle$, which implies a contradiction with above, as

   $$\langle\psi_2|E_2|\psi_2\rangle = |\beta|^2\langle\phi|E_2|\phi\rangle \leq |\beta|^2 < 1,$$

   where the second last inequality follows from the observation that

   $$\langle\phi|E_2|\phi\rangle \leq \sum_i \langle\phi|E_i|\phi\rangle = \langle\phi|\phi\rangle = 1.$$

2.2.5 Projective measurements

Projective measurements is a special case of the general measurement postulate. Projective measurements actually turn out to be *equivalent* to the general measurement postulate, when they are augmented with the ability to perform unitary transformations.

Projective measurements: A projective measurement is described by an *observable*, M, a Hermitian operator on the state space of the system being observed.

$$M = \sum_m m P_m$$

(spectrally decomposed), where P_m is the projector onto the eigenspace of M with eigenvalue m.
The probability of getting result m of the state $|\psi\rangle$ is given by

$$p(m) = \langle\psi|P_m|\psi\rangle.$$

Given that outcome m occurred, the state of the quantum system immediately after the measurement is

$$\frac{P_m|\psi\rangle}{\sqrt{p(m)}}$$

If $\sum_m M_m^\dagger M_m = I$, the M_m are Hermitian, and $M_m M_{m'} = \delta_{m,m'}$, then Postulate 3 reduces to a projective measurement as just defined.

It is very easy to calculate average values for projective measurements. By definition, the average value of the measurement is

$$\begin{aligned}
\mathbf{E}(M) &= \sum_m m p(m) \\
&= \sum_m m\langle\psi|P_m|\psi\rangle \\
&= \langle\psi|\left(\sum_m m P_m\right)|\psi\rangle \\
&= \langle\psi|M|\psi\rangle \\
&= \langle M\rangle
\end{aligned}$$

The standard deviation associated to observations of M,

$$\begin{aligned}
[\Delta(M)]^2 &= \langle(M - \langle M\rangle)^2\rangle \\
&= \langle M^2\rangle - \langle M\rangle^2.
\end{aligned}$$

For large # limit $\Delta(M) = \sqrt{\langle M^2\rangle - \langle M\rangle^2}$. This formulation of measurement and standard deviations in terms of observables gives rise in an elegant way to results such as the *Heisenberg uncertainty principle*.

Two widely used nomenclatures for measurements deserve emphasis. Rather than giving an observable to describe a projective measurement, often people simply list a complete set of orthogonal projectors P_m

satisfying the relations $\sum_m P_m = I$ and $P_m P_{m'} = \delta_{m,m'} P_m$. The corresponding observable implicit in this usage is $M = \sum_m m P_m$. Another widely used phrase, to 'measure in a basis $|m\rangle$', simply means to perform the projective measurements with $|m\rangle\langle m|$.

eg. examples of projective measurements on single qubits.

Measurement of Z on the state $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ gives result $+1$ with probability $\langle\psi|0\rangle\langle 0|\psi\rangle = 1/2$, similarly result -1 with probability 1/2.

Generally, suppose $\vec{v}$ is any real three-dimensional unit vector. Then we can define and observable:

$$\vec{v} \cdot \vec{\sigma} \equiv v_1\sigma_1 + v_2\sigma_2 + v_3\sigma_3.$$

Measurement of this observable is sometimes referred to as a 'measurement of spin along the $\vec{v}$ axis'.

The uncertainty principle
A, B : Hermitian operators.
$|\psi\rangle$: a quantum state.
Suppose $\langle\psi|AB|\psi\rangle = x + iy$
$\langle\psi|[A, B]|\psi\rangle = 2iy$
$\langle\psi|\{A, B\}|\psi\rangle = 2x$
This implies that

$$|\langle\psi|[A, B]|\psi\rangle|^2 + |\langle\psi|\{A, B\}|\psi\rangle|^2 = 4|\langle\psi|AB|\psi\rangle|^2$$

By the Cauchy–Schwarz inequality

$$|\langle\psi|AB|\psi\rangle|^2 \leq \langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle,$$

with above equation and dropping anti-commutation term gives

$$|\langle\psi|[A, B]|\psi\rangle|^2 \leq 4\langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle.$$

Suppose C and D are two observables. Substituting $A = C - \langle C\rangle$ and $B = D - \langle D\rangle$ into the last equation, we contain Heisenberg's uncertainty principle as it is usually states:

$$\Delta(C)\Delta(D) \geq \frac{|\langle\psi|[C, D]|\psi\rangle|}{2}$$

A common misconception about the uncertainty principle:
measuring an observable C to some 'accuracy'$\Delta(C)$ causes the value of D to be 'disturbed' by an amount $\Delta(D)$ in such a way that some sort of inequality similar to above relation is satisfied.While it is true that measurements in quantum mechanics cause disturbance to the system being measured, this is most emphatically *not* the content of the uncertainty principle.
The correct interpretation of the uncertainty principle is that if we prepare a large number of quantum systems in identical states, $|\psi\rangle$, and then perform measurements of C on some of those systems, and of D in others,then the standard deviation $\Delta(C)$ of the C results times the standard deviation $\Delta(D)$ of the results for D will satisfy the inequality relation.
eg. Consider the observables X, Y when measured for the quantum state $|0\rangle$.

$$\Delta(X)\Delta(Y) \geq \frac{|\langle 0|[X, Y|0\rangle|}{2} = \langle 0|Z|0\rangle = 1.$$

One elementary consequence of this is that $\Delta(X)$ and $\Delta(Y)$ must both be strictly greater than 0, as can be verified by direct calculation.

2.2.6 POVM measurement

Review of Postulate 3 :

1. It gives a rule describing the measurement statistics, that is, the respective probabilities of the different possible measurement outcomes.

2. it gives a rule describing the post measurement state of the system.

However, for some application post-measurement state is of little interest, with the main item of interest being the probabilities of the respective measurement outcomes.

This is the case in an experiment where the system is measured only once, upon conclusion of the experiment. In such instances there is a mathematical tool known as the it Positive Operator-valued Measurement (POVM) formalism

Suppose a measurement described by measurement operators M_m is performed upon a quantum system in the state $|\psi\rangle$. Then the probability of outcome m is given by $p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$. Define

$$E_m \equiv M_m^\dagger M_m$$

E_m is positive operator such that $\sum_m E_m = I$ and $p(m) = \langle\psi|E_m|\psi\rangle$. Thus the set of E_m are sufficient of determine the probabilities of the different measurement outcomes.

E_m : **POVM** elements
$\{E_m\}$: **POVM**

If a projective measurement satisfies $P_m P_{m'} = \delta_{m,m'} P_m$ and $\sum_m P_m = I$, then all the POVM elements are the same as the measurement operators themselves, since $E_m \equiv P_m^\dagger P_m = P_m$.

Suppose that $\{E_m\}$ is some arbitrary set of positive operators such that $\sum_m E_m = I$. Defining $M_m \equiv \sqrt{E_m}$, so that $\sum_m M_m^\dagger M_m = \sum_m E_m = I$, and therefore the set $\{M_m\}$ describes a measurement with POVM $\{E_m\}$.

For this reason it is convenient to *define* a POVM to be any set of operators $\{E_m\}$ such that:

1. each operator E_m is positive
2. the *completeness relation* $\sum_m E_m = I$ is obeyed
3. the probability of outcome m is given by $p(m) = \langle\psi|E_m|\psi\rangle$

The other example of POVM formalism as a guide for our intuition in quantum computation and quantum information.

Suppose Alice gives Bob a qubit prepared in one of two states $|\psi_i\rangle = |0\rangle$ or $|\psi_2\rangle = |+\rangle$. It is important to Bob to determine whether he has been given $|\psi_1\rangle$ or $|\psi_2\rangle$ with perfect reliability. However it is possible for him to perform a measurement which distinguishes the states some of the time, but *never* makes an error of mis-identification. Consider a POVM containing three elements,

$$\begin{aligned} E_1 &\equiv \frac{\sqrt{2}}{1+\sqrt{2}}|1\rangle\langle 1|, \\ E_2 &\equiv \frac{\sqrt{2}}{1+\sqrt{2}}|-\rangle\langle -|, \\ E_3 &\equiv I - E_1 - E_2. \end{aligned}$$

Bob has $E_1 \to |\psi_2\rangle$
Bob has $E_2 \to |\psi_1\rangle$
However, the case of $E_3 \to$ can not identify the state he have been given.
The key point is that Bob *never* makes a mistake identifying the state he has been given. This infallibility comes at the price that sometimes Bob obtains no information about the identity of the state.

General measurements, projective measurements, and POVMs
Projective measurements : useful for the region of coarsely measurements are meaningful.
In quantum computation and quantum information we aim for an exquisite level of control over the measurements that may be done, and consequently it helps to use a more comprehensive formalism for the description of measurements.
Of course projective measurements augmented by unitary operations turn out to be completely *equivalent* to general measurements.
Why we start with the general formalism

1. General measurements are in some sense mathematically simpler than projective measurements, since they involve fewer restrictions on the measurement operators. This simpler structure also gives rise to many useful properties for general measurements that are not possessed by projective measurements.

2. There are important problems in quantum computation and quantum information – such as the optimal way to distinguish a set of quantum states – the answer to which involves a general measurement, rather than a projective measurement.

3. Projective measurements are repeatable in the sense that if we perform a projective measurement once, and obtain the outcome m, repeating the measurement gives the outcome m again and does not change the state. After a measurement, $|\psi_m\rangle = (P_m|\psi\rangle)/\sqrt{\langle\psi|P_m|\psi\rangle}$ and repeating P_m

2.2.7 Phase

$|\psi\rangle$ and $e^{i\theta}|\psi\rangle$: Two states are same, up to the global phase factor $e^{i\theta}$.

$$\because \langle\psi|e^{-i\theta}M_m^\dagger M_m e^{i\theta}|\psi\rangle = \langle\psi|M_m^\dagger M_m|\psi\rangle$$

Therefore observational point of view these two states are identical.

Consider the states

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad \text{and} \qquad \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

In the first state the amplitude of $|0\rangle$ is $1/\sqrt{2}$. For the second state the amplitude is $-1/\sqrt{2}$. In each case the *magnitude* of the amplitudes is the same, but they differ in sign. More generally,

$$|\psi\rangle = a|0\rangle + b|1\rangle \qquad (a = e^{i\theta}b),$$

we say that two amplitudes *differ by a relative phase.* More generally still, two states are said to *differ by a relative phase* in some basis if each of the amplitudes in that basis is related by such a phase factor. $|+\rangle$ and

$|-\rangle$ are the same up to a relative phase shift because the $|0\rangle$ amplitudes are identical, and the $|1\rangle$ amplitudes differ only by a relative phase factor of -1

For relative phase the phase factors may vary from amplitude to amplitude. This makes the relative phase a basis–dependent concept unlike global phase. As a result, states which differ only by relative phases in some basis give rise to physically observable differences in measurement statistics, and it is not possible to regard these states as physically equivalent.

2.2.8 Composite systems

> **Postulate 4**: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n, and system number i is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots |\psi_n\rangle$.

Why? We certainly expect that there be *some canonical way* of describing composite systems in quantum mechanics. The *superposition principle of quantum mechanics*, so called, could be expected. (One expect the linearity) This is <u>not a derivation</u>, since we are not taking the superposition principle as a fundamental part of our description of quantum mechanics.

The proof of the statement, that is projective measurements together with unitary dynamics are sufficient to implement a general measurement, makes use of composite quantum systems.

Suppose a quantum system with state space Q, measurement operators M_m on Q. Introduce *ancilla system*, with state space M, having a orthonormal basis $|m\rangle$.
The ancilla system can be regarded as merely mathematical device appearing in the construction, or it can be interpreted physically as an extra quantum system introduced into the problem.

$|0\rangle$: any fixed state of M, define an operator U on products $|\psi\rangle|0\rangle$ of states $|\psi\rangle$ from Q with the state $|0\rangle$ by

$$U|\psi\rangle|0\rangle \equiv \sum_m M_m|\psi\rangle|m\rangle.$$

Using the orthonormality of the states $|m\rangle$ and the completeness relation $\sum_m M_m^\dagger M_m = I$, U preserves inner products between states of the form $|\psi\rangle|0\rangle$,

$$\begin{aligned}
\langle\phi|\langle 0|U^\dagger U|\psi\rangle|0\rangle &= \sum_{m,m'} \langle\phi|M_m^\dagger M_m|\psi\rangle\langle m|m'\rangle \\
&= \sum_m \langle\phi|M_m^\dagger M_m|\psi\rangle \\
&= \langle\phi|\psi\rangle
\end{aligned}$$

Probability $p(m)$ of projectors $P_m \equiv I_Q \otimes |m\rangle\langle m|$ is

$$p(m) = \langle\phi|\langle 0|U^\dagger P_m U|\psi\rangle|0\rangle$$

$$= \sum_{m'm''} \langle\psi|M^\dagger_{m'}\langle m'|(I_Q \otimes |m\rangle\langle m|)M_{m''}|\psi\rangle|m''\rangle$$
$$= \langle\psi|M^\dagger_m M_m|\psi\rangle.$$

The joint state of the system QM after measurement, conditional on result m occurring, is given by

$$\frac{P_m U|\psi\rangle|0\rangle}{\sqrt{\langle\psi|U^\dagger P_m U|\psi\rangle}} = \frac{M_m|\psi\rangle|m\rangle}{\sqrt{\langle\psi|M^\dagger_m M_m|\psi\rangle}}$$

It follows that the state of system M after the measurement is $|m\rangle$, and the state of system Q is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M^\dagger_m M_m|\psi\rangle}}$$

Thus unitary dynamics, projective measurements, and the ability to introduce ancillary systems, together allow any measurement of the form described in Postulate 3 to be realized.

Postulate 4 also enables us to define one of the most interesting and puzzling ideas associated with composite quantum systems - *entanglement.*

$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

This state hase the remarkable property that there are no single qubit states $|a\rangle$ and $|b\rangle$ such that $|\psi\rangle = |a\rangle|b\rangle$.

We say that a state of a coposite system having this property (that is can't be written as a product of states of its component systems) is an *entangled* state. For reasons which nobody fully understands, entangled states play a crucial role in quantum computation and quantum information. (Teleportation, dense coding, Bell's inequality)

2.2.9 Quantum mechanics: a global view

Review :

- Postulate 1 sets the arena for quantum mechanics, by specifying how the state of an isolated quantum system is to be described.
- Postulate 2 tells us that the dynamics of evolution.
- Postulate 3 tells us how to extract information from our quantum system by giving a prescription for the desription of measurement.
- Postulate 4 tells us how the state spaces of different quantum systems may be combined to give a description of the composition system.

Odd stuff :

- We cannot directly observe the state vector.
- Classical physics tells us that the fundamental properties of an object, like energy, position, and velocity, are directly accessible to observation. $\rightarrow$ are not fundamental quantities in quantum mechanics.
  It is as thought there is a *hidden world* in quantum mechanics, which we can only indirectly and imperfectly access.
- Observation in quantum mechanics is an invasive procedure that typically changes the state of the system.